MAGDALENE COLLEGE

NATURAL SCIENCES ADMISSIONS SAMPLE TEST 2016

This is a truncated sample admissions test demonstrating the style; in the actual exam, there will be a larger selection of questions to choose from. We know that candidates have different areas of knowledge, and are not always taught in the same sequence. The aim is therefore to demonstrate what candidates can do, rather than to trip them up with what they may not yet know. For this reason, the test focuses on straightforward science, but in unfamiliar contexts.

Time allowed: 1 hour (+ 15 minutes to prepare)
The first 15 minutes are for reading only; you will be told when you may begin to write.

Before you begin, read these instructions carefully:

The paper has three sections, A, B and C.
Candidates for Physical Natural Sciences must attempt Section A and Section B.
Candidates for Biological Natural Sciences must attempt Section A and Section C.

Begin each section on a separate sheet of paper.

You are recommended to spend about 30 minutes on each section.

We are aware that you have different backgrounds, and that some of the material in each section may not be familiar to you. Select those questions which you can do, completing as many as you can. You are not expected to be able to complete them all in the time available.

The use of calculators is not permitted
Use estimates and approximations where needed: this is not a test of arithmetic.
SECTION A: MATHEMATICS

INSTRUCTIONS:
Spend 30 minutes on this section. Complete as many questions as you can, in any order. You are not expected to be able to complete them all in the time available. Calculators are not permitted. Use estimates and approximations where needed. Be concise.

1. Calculate:
   (a) $101^2 - 99^2$
   (b) $\log_{10} 1000$
   (c) $\frac{d}{dx} (3x^2 + 5)^{27}$
   (d) $\int_{0}^{10} (x^3 + x)dx$
   (e) $e^{i\pi/6}$
   [5 marks]

2. A fair coin is flipped $N$ times. What is the probability of all of the results being heads?
   You become suspicious. How many consecutive heads would you have to observe before concluding that the coin is, in fact, biased? [4 marks]
   [Hint: Use a 2-tailed test at the 5% significance level.]

3. You have 100 m of fencing to enclose a rectangular field, with dimensions $h$ and $w$. Find the maximum area you can enclose. If your field need not be rectangular, what shape should you choose? [4 marks]

4. The two variables, $x$ and $y$ satisfy:
   $$\frac{d^2y}{dx^2} + ky = 0.$$  
   Furthermore, $y = 0$ when $x = 0$. State the solution of this equation. [4 marks]
   [You do not need to derive your solution in full.]

5. Estimate (to within 1 order of magnitude) how many golf-balls are sold per year worldwide? [4 marks]
   [Hint: make estimates of the quantities you know, e.g. fraction of people who play golf.]
SECTION B: PHYSICAL NATURAL SCIENCES

INSTRUCTIONS:
Spend 30 minutes on this section. Complete as many questions as you can, in any order. You are not expected to complete all of them in the time. Calculators are not permitted. Use estimates and approximations where needed, for example \( \pi \approx 3 \approx \sqrt{10} \). Numerical answers may be given to 1 significant figure or as fractions. Be concise.

1. A party trick involves filling a bucket with water, and swinging it over your head. Assuming you wish to remain dry, how fast must you swing the bucket? \[2 \text{ marks}\]
   [Radius of arm + bucket + handle = 1 m.]

2. A 100 kg person can safely land after jumping off a wall 5 m high. How fast is this? A parachutist must descend with a terminal velocity below this speed. What area of parachute does he need? Assume that the parachute perfectly sweeps up all the air in its path. \[5 \text{ marks}\]
   [Density of air \( \approx 1 \text{ kg m}^{-3} \).]

3. The Voyager 1 space probe is powered by a Radioisotope Thermoelectric Generator (using the heat from the \( \alpha \)-decay of Plutonium). It was launched in 1977, with an initial power availability of 500 W. It needs at least 125 W to operate usefully. How long could it last for? Explain (in a few lines) how an RTG works. \[5 \text{ marks}\]
   \([^{238}\text{Pu half-life} = 88 \text{ years}].\]

4. The International Space Station orbits just above the atmosphere every 90 minutes. Use this to calculate the radius of the Earth.

   Now, considering the Earth as an isolated, charged sphere, what is the capacitance of the Earth.

   Comment on your answer. \[7 \text{ marks}\]
   \([\text{In the ISS’s low-earth-orbit, } g_{\text{orbit}} \approx g_{\text{surface}} \cdot \epsilon_0 = 8.9 \times 10^{-12} \text{ F/m.}]\]

5. The Lorentz Gamma-factor in Relativity is given by:

   \[
   \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.
   \]

   Sketch a graph of \( \gamma \) against \( v \). \[3 \text{ marks}\]
SECTION C: BIOLOGICAL NATURAL SCIENCES

INSTRUCTIONS:
Spend 30 minutes on this section.
Write an essay in response to one of the three following questions.

1. Explain why an understanding of molecular structure is important to medicine.

2. What, in your opinion, is the most important advance in biological science in the past 50 years?